milk for at least three days, and a smaller degree of infection for ten days or even longer. At the same time he shows that blow-flies produce gross infection for six to nine days with non-spore-bearing micro-organisms and some degree of infection for three or four weeks. The investigator thinks that it is probable, at any rate in the later stages, that infection is mainly due either to direct infection with the crop contents vomited through the proboscis, or to direct infection by means of the limbs which have been reinfected with vomited material.

These experiments were so conducted as to afford no information as to the extent to which house flies bred from larvæ fed on naturally infected excreta and similar materials are apt themselves to be infected.

L. O. HOWARD

Lectures on Fundamental Concepts of Algebra and Geometry. By J. W. YOUNG. Prepared for publication with the cooperation of W. W. DENTON, with a note on the growth of algebraic symbolism by W. G. MITCHELL. Pp. vii + 247. New York, The Macmillan Company. 1911.

While the teacher of secondary mathematics finds a large amount of English literature on the teaching of his subject he looks in vain for much that is well adapted to give him a deep insight into the fundamental theory of the subjects with which he has to deal. The English language contains no encyclopedia on elementary mathematics like Weber and Wellstein's "Enzyklopädie der Elementarmathematik," or like the new Italian encyclopedia which is being prepared. It has no histories like Cantor's or even like Tropfke's. It has no periodical like *L'Enseignement Mathématique*, and no large mathematical encyclopedias like the great works which are now being published in German and in French.

Although the small size of the book under review precludes any hopes that we might have here a work to which the teacher of secondary mathematics may turn for an answer to most of his questions, yet he will find here an unusually clear exposition of a large number of things relating to the logical foundation of algebra and geometry. The brevity of the exposition will doubtless be welcomed by many who are looking for a first general survey of some basic matters, and it is to be hoped that they may become sufficiently interested to pursue the thoughts further, as they are encouraged to do by a fair number of references.

The book is modern in spirit, and, to a large extent also, in subject matter. Considerable attention is given to historical settings but the logical element receives the greatest emphasis. It opens up view points which are of great interest even if they may not always be acceptable to the reader. From the nature of the case many of the questions treated are such as to give rise to different views, but their fundamental importance justifies inquiries even if these do not always receive a complete answer. One of the most important lessons for the young mathematician to learn is a keen realization of the narrow limits of the explored parts of mathematics as compared with those regions which invite our inquiry and baffle our efforts.

The contents of the volume can be readily inferred, in the main, from its title. After a brief consideration of Euclid's elements and non-euclidean geometry, the author considers the logical significance of definitions, axioms and postulates, the consistency, independence and categoricalness of a set of assumptions. This is followed by a consideration of the fundamental notions of class, correspondence and group, and the development of the concepts of real and complex numbers. It is pointed out that from the abstract point of view algebra and geometry are identical in the sense that each includes the other, and that this explains the interrelations between these subjects.

On page 194 the author repeats a historical error which is very wide spread in mathematical literature, as regards the early use of the term function for integral power of a variable. This error seems to have been started by d'Alembert and it has been re-

peated by a large number of prominent mathematicians.[1] That the graph on page 214 is the graphic representation of the function in question is open to serious doubts, which should not have been passed over in a work on logical foundations. The statement on page 101 that "Diophantus of Alexandria. who lived 300 A.D., seems to have been the first actually to have made use of rational numbers" is apt to mislead the reader even if a footnote helps to ascertain the author's meaning. Taken by itself this statement seems absurd.

These are, however, matters of secondary importance and the book under review seems to be remarkably free from errors if we consider its wide scope. In particular, the proof seems to have been read with unusual care and one can only wish that the book will be very widely read, especially by those who are preparing to teach secondary mathematics. Its style is attractive and many of the questions which it treats are so far reaching that one may reasonably expect that it will find a considerable number of readers outside of the circle of professional mathematicians.

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[1] Cf. "Encyclopédie des Sciences Mathematiques," Tome 2, Vol. 1, p. 3; Cantor's "Geschichte," Vol. 3, 1901, pp. 215, 456–7.

SPECIAL ARTICLES

COLOR DISPERSION IN THE ASTIGMATIC EYE

When an astigmatic eye views a bright point of light in which only the rays near the ends of the visible spectrum are present, the image of the source is blurred by fringes or wings of red and blue. If the eye has a well-defined axis of astigmatism but is otherwise fairly emmetropic, the appearance of such a source is so curious as to compel attention. The purplish image is then crossed by a pronounced red band parallel to that meridian of the eye in which the curvature is least, and by a blue band at right angles to it. In the case of astigmatism with the rule, the red band is approximately horizontal. The experiment is easily tried in a darkened room by allowing the light from a flame or electric lamp to pass through a hole a few millimeters in diameter in a screen, or better by placing the lamp in a box having a small hole in one side. One or two thicknesses of common "pot blue" glass are placed over the opening, which is then viewed from a distance of two meters or more. The blue glass, as is well known, is fairly transparent to red light. Distant blue lights seen at night, such as the "dwarf signals" in railroad yards, show the effect well.[1] Indeed, the appearance can be seen by viewing any bright light through a blue glass held in front of the eye. A person free from astigmatism can see the effect by holding a cylindrical lens in front of the eye.

The explanation is simple, and has very likely occurred to many who have noticed the effect. However, the writer has been unable to find any reference to it, either in the classical memoirs of Helmholtz and his predecessors, or in such later writings as he has access to. Astigmatic vision seems to have been considered only on the tacit assumption that dispersion could be neglected—an assumption that is sufficient with ordinary white light, in which the yellow and green rays predominate in determining our visual sensations. It is only when these intermediate rays are excluded that the effects of dispersion become noticeable.[2]

Taking the type of astigmatism most commonly found, let us assume that the radius of curvature of the cornea is less for the vertical than for the horizontal meridian. If the eye observes a distant point-source giving only

[1] Among those whom the writer asked whether they had noticed the crossed red and blue bands was a certain railroad employé, who not only observed the appearance to a marked degree, but also volunteered the explanation that the dwarf signal "had a dirty glass." Remarks of this sort show how unconscious we are of our own defects of vision.

[2] For example, Helmholtz describes some interesting experiments on the effects of chromatic dispersion in the eye; these are also recounted by Lummer in Müller-Pouillet's "Lehrbuch der Physik." In all these cases the eye is assumed to be free from astigmatic defects.